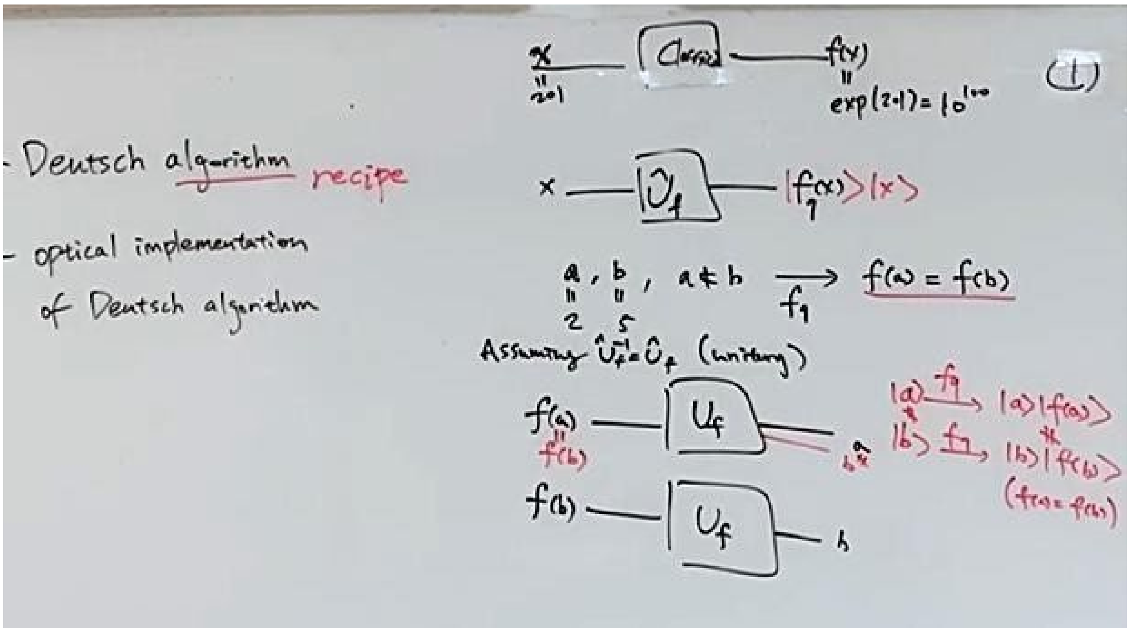
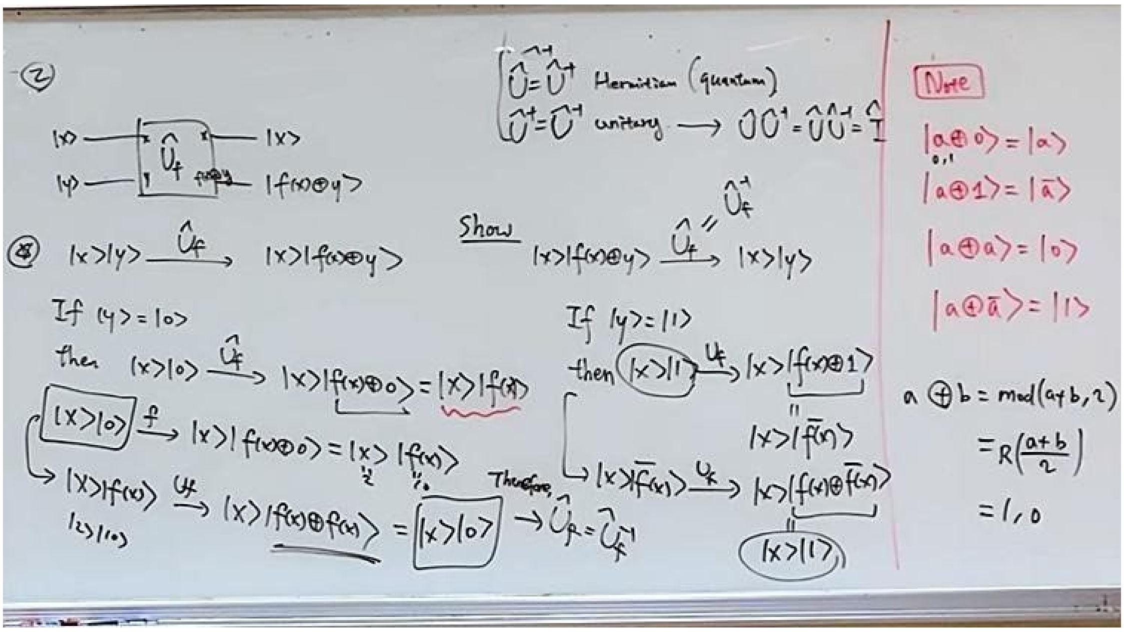
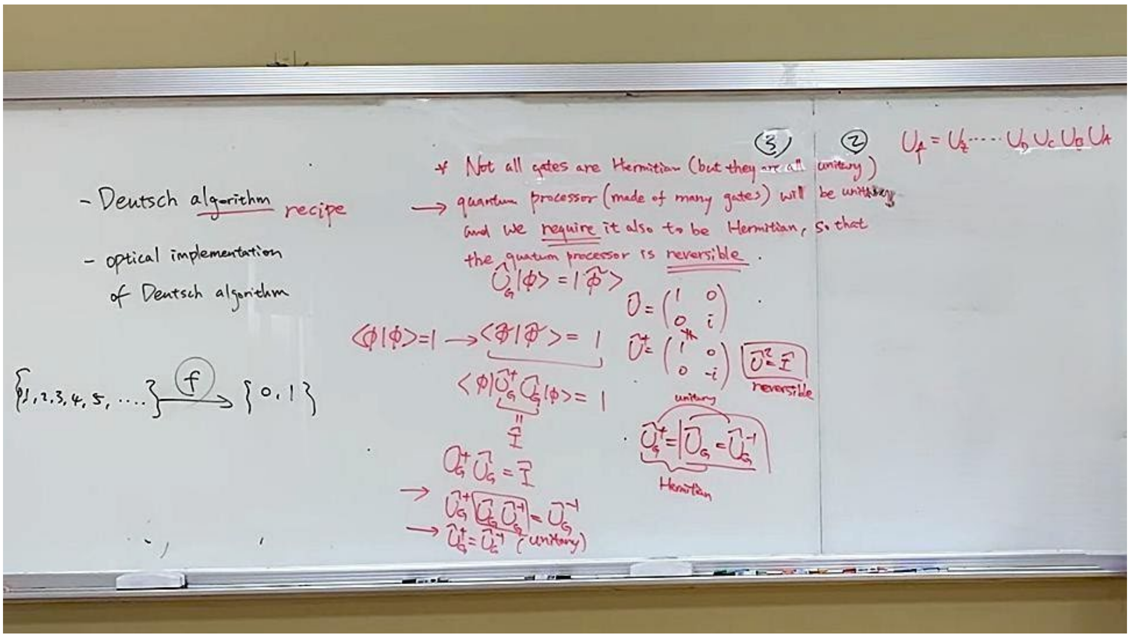
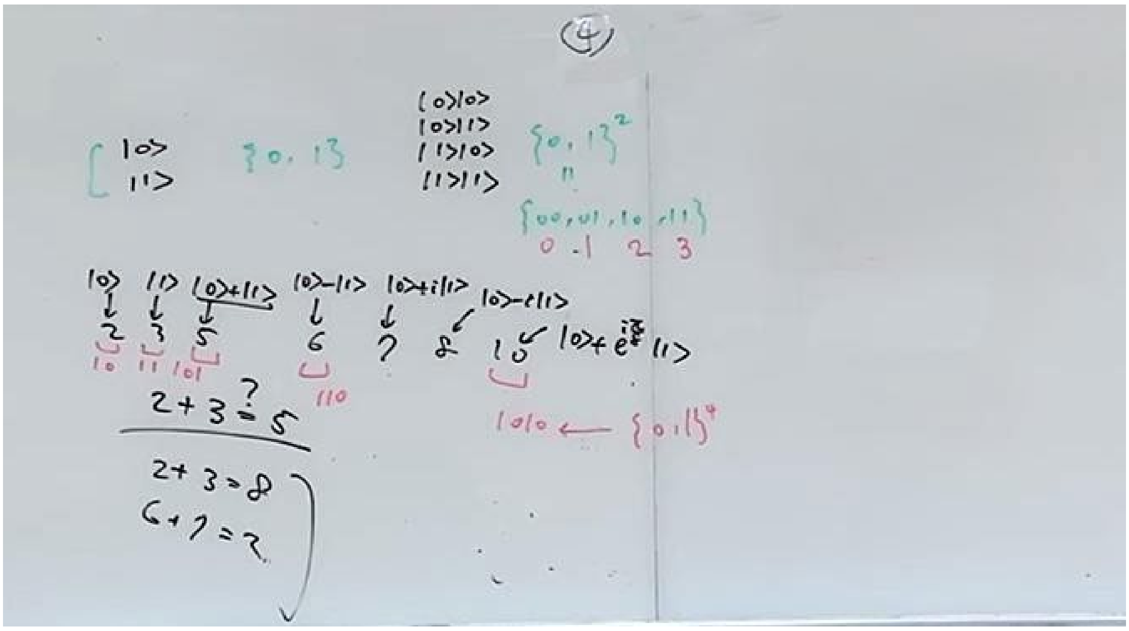
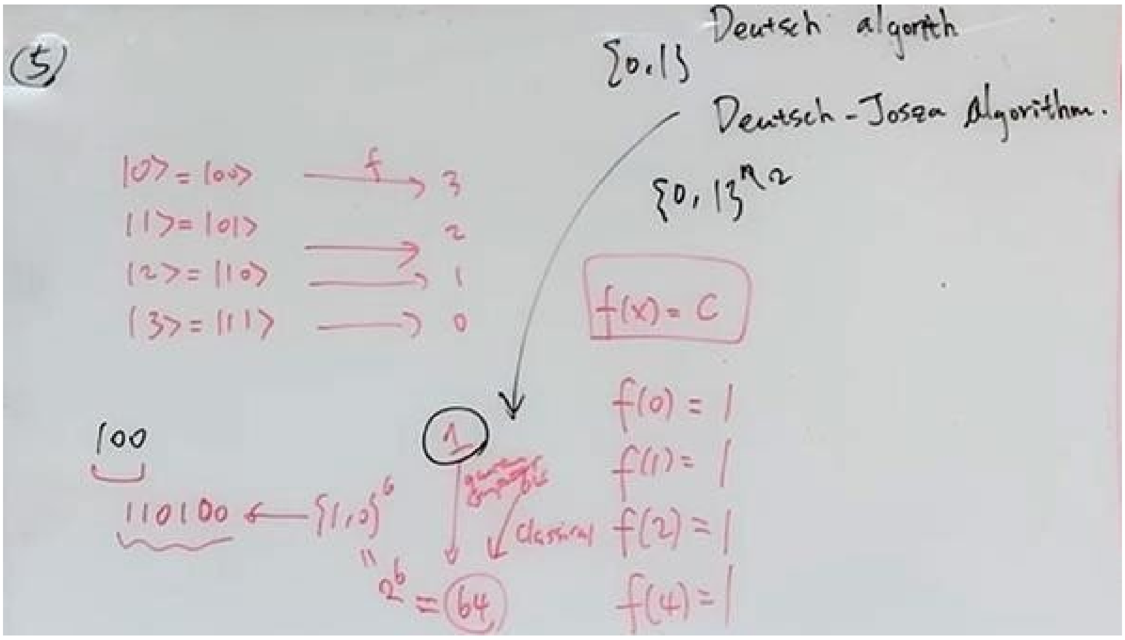
Lecture13 Deutsch algorithm

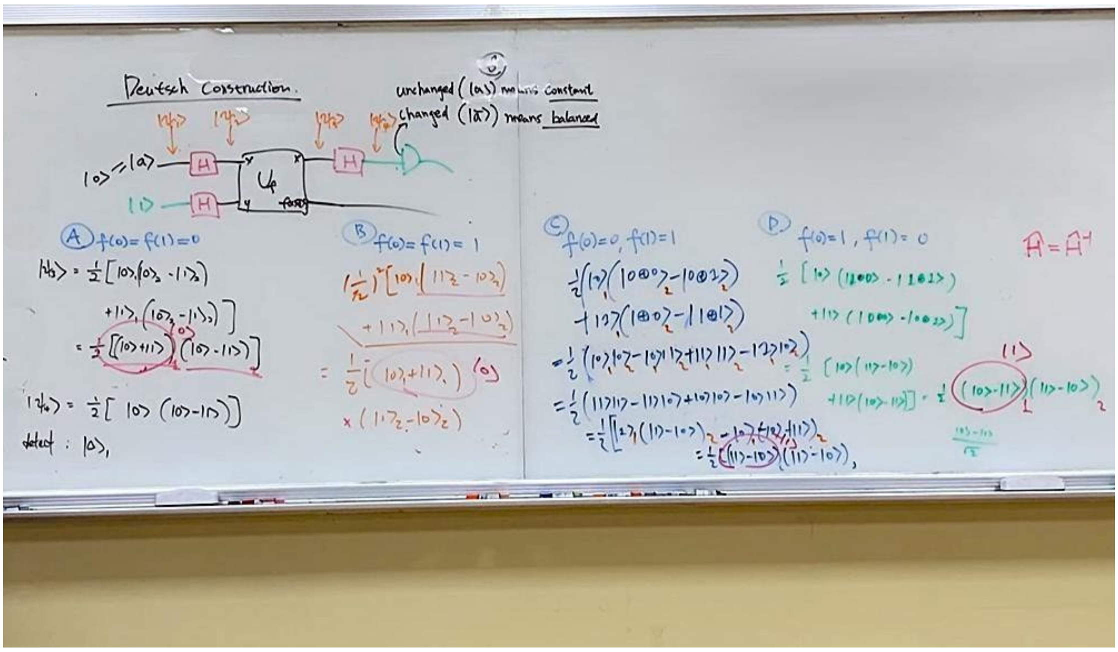


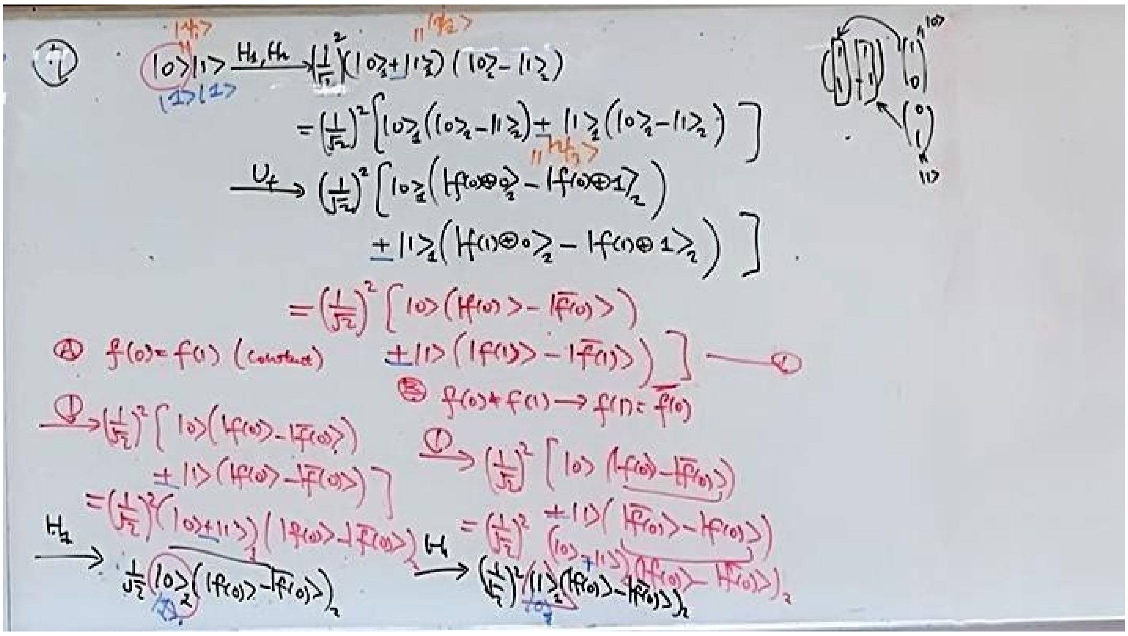


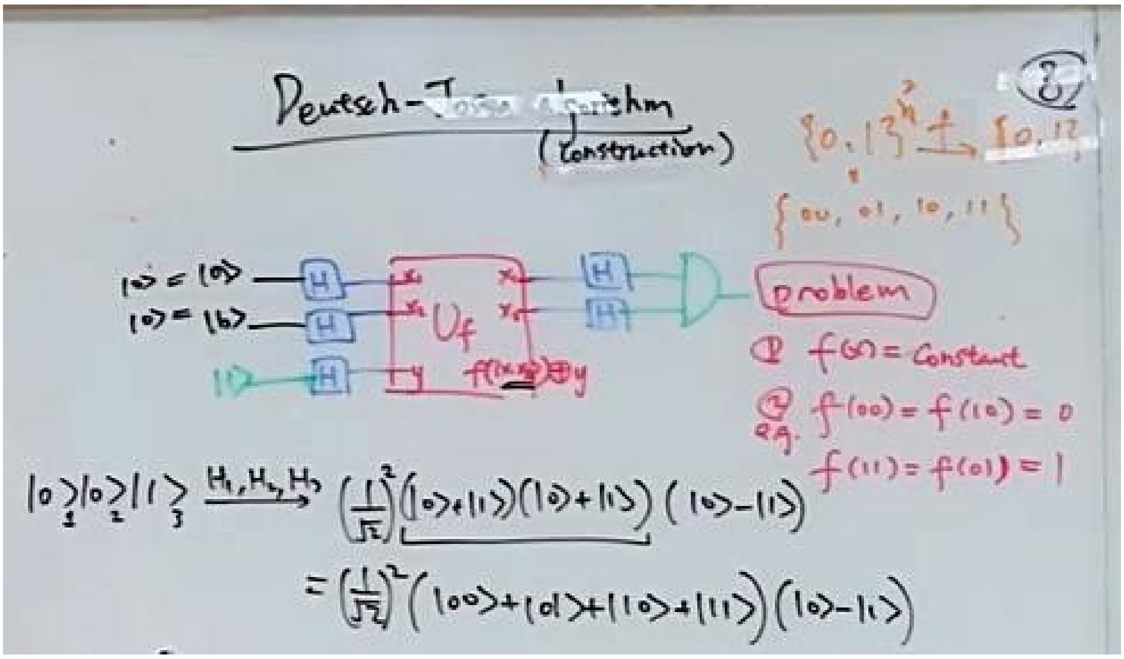

- Deutsch algorithm recipe
- optical implementation of Deutsch algorithm
{1,2,3,4,5, ...} f {0,1}
* Not all gates are Hermitian (but they are all unitary)
→ quantum processor (made of many gates) will be unitary
and we require it also to be Hermitian, so that
the quantum processor is reversible.
unitary
reversible
Hermitian
(unitary)

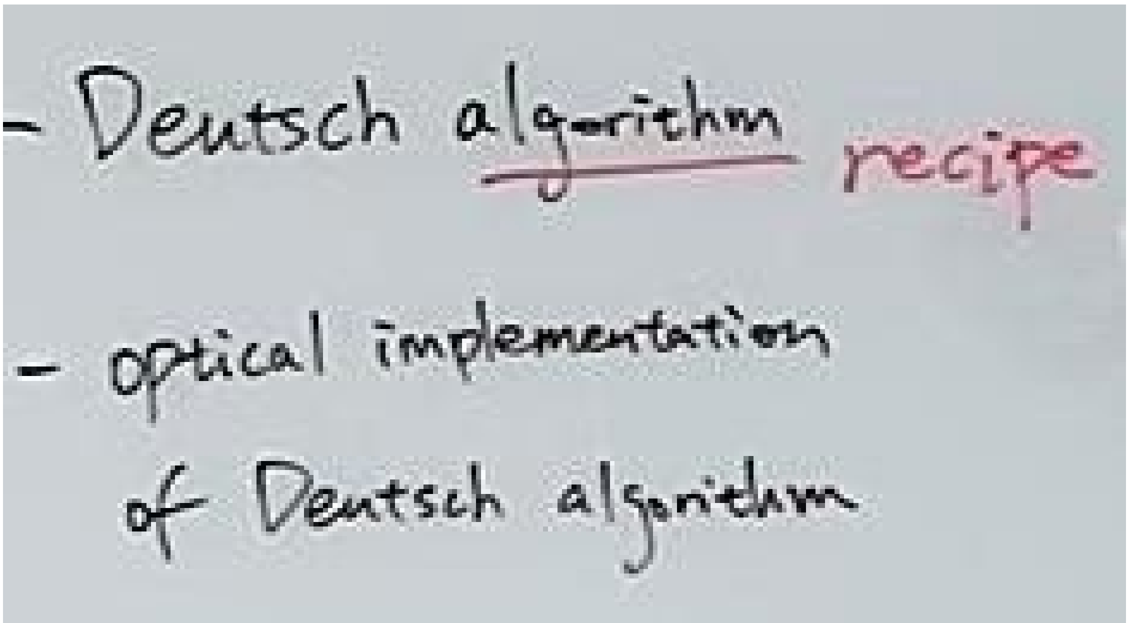

|0>
|1>
{0, 1}
|0>|0>
|0>|1>
|1>|0>
|1>|1>
{00, 01, 10, 11}
0 1 2 3
|0> |1> |0>+|1> |0>-|1> |0>+i|1> |0>-i|1>
2 3 5 6 7 8 10
10 11 101 110
1010
2+3=5
2+3=8
6+7=?


Deutsch algorith
Deutsch-Josza Algorithm.
f(x) = C
f(0) = 1
f(1) = 1
f(2) = 1
f(4) = 1
classical
100
110100
64


Deutsch Construction.
unchanged (|a>) means constant
changed (|ā>) means balanced
f(0)=f(1)=0
f(0)=f(1)= 1
f(0)=0, f(1)=1
f(0)=1, f(1)=0
detect : |0>1


|0>|1> H1,H2 → (1/√2)^2 (|0>1+|1>1)(|0>2−|1>2)
= (1/√2)^2 [|0>1(|0>2−|1>2) ± |1>1(|0>2−|1>2)]
Uf → (1/√2)^2 [|0>1(|f(0)⊕0>2 − |f(0)⊕1>2)
± |1>1(|f(1)⊕0>2 − |f(1)⊕1>2)]
= (1/√2)^2 [|0>(|f(0)> − |f̄(0)>)
± |1>(|f(1)> − |f̄(1)>)]
A f(0)=f(1) (constant)
B f(0)≠f(1) → f(1)=f̄(0)
H2
H


Deutsch-Jozsa algorithm
(construction)
{0,1}^n → {0,1}
{00, 01, 10, 11}
|0>=|0>
|0>=|0>
|1>
H
Uf
f(x1,x2)⊕y
problem
① f(x)= Constant
② f(00)=f(10)=0
e.g. f(11)=f(01)=1
|0>1|0>2|1>3 H1,H2,H3 → (1/√2)^2 (|0>+|1>)(|0>+|1>)(|0>−|1>)
= (1/√2)^2 (|00>+|01>+|10>+|11>)(|0>−|1>)


- Deutsch algorithm recipe
- optical implementation
of Deutsch algorithm